

# Optics

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## General

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- These notes are adapted from the lecture notes

## Part 1 – Data Storage and Music Reproduction

### The introduction of the CD player

- CD has higher dynamic range than vinyl
- DVD has 7x the data density of the CD

### The CD pick-up as an optical system

- Stimulated emission in laser diode
- Multibeam diffraction from grating
- Polarisation in polarising beam splitter
- Retardance in  $\frac{1}{4}$  wave plate
- Lens systems and diffraction in the objective lens
- Interference and encoding at the disc surface

### The light source – the laser and stimulated emission

#### Why a laser?

- Need to be focused onto a very small spot ( $1\mu\text{m}$ )
- High SNR
- Monochromatic

#### Stimulated and spontaneous emission

- Spontaneous emission
  - Excited atom emits photon in a random manner but with a characteristic lifetime determined by the strength of the oscillator dipole moment between the excited and relaxed state of the atom.
- Stimulated emission
  - Does not emit light until another photon interacts with the atom.
  - The emitted photon has the same properties of the stimulating photon.

#### Population inversion

- More atoms in an excited state than the lower lying states, as opposed to thermally distributed

- This condition must be satisfied for stimulated emission
- Atoms may be selectively pumped into a higher lying state
  - Optical pumping (Ruby laser)
  - Gas collisions (HeNe laser)
  - Electrical pumping (Diode laser)

### Gain vs loss

- Build up stimulated emission using reflective mirrors called cavity or resonator
- Gain of the medium must be higher than the transmission of the mirrors
- Light is allowed to escape by making one mirror partially reflective, to provide a useful light beam.

### Laser characteristics

- Highly temporal coherent – light waveform does not vary much over time
  - Width of spectrum is narrow
- Highly spatially coherent – different parts of the wavefront are correlated to each other
  - Able to be focused onto small spots
- Light emitted is of same wavelength, with a stable phase, in the same direction with the same polarisation.

### Homework Problem:

1. A Laser has a threshold at 10mA pump current and a slope efficiency of 0.5mW output per mA of pump current.

a) Plot the Output Characteristic.

b) The reflectivities of the laser mirrors have been chosen for maximum output power. In what way could you modify the mirrors to lower the threshold?

### The semiconductor diode laser

- External potential applied to the p-n junction, creating an inversion of electron-hole pairs.
- In the presence of a stimulating photon, the electrons in the interface recombine with holes in the valence band to emit a clone of the photon of light.
- Efficiency > 50% with typical output power in the order of mW.

## Optical extraction of the information

### The CD surface and encoding

- Pits / Land – 0
- Transition – 1
- Data encoded using “eight to fourteen” 1 byte uses 14 bits, so that:
  - No 2 ones can occur consecutively
  - Never < 3 or > 10 zeros in a row

### Expression for a propagating wave

- Frequency  $\nu$ , wavelength  $\lambda$ , phase offset  $\phi$ , amplitude of electric field vector describing the wave  $A$

$$E(t, x) = A \sin\left(2\pi\nu t - \frac{2\pi x}{\lambda} + \phi\right)$$

- The electric field oscillates too fast for the photodetector to be measured directly.
  - Measure the average intensity instead:

$$I(x, t) \propto \overline{E(t, x)E(t, x)}$$

$$I(t, x) \propto \frac{A^2}{2}$$

### Speed of light and refractive index, frequency & wavelength

- Period  $\tau$  – time taken for one wavelength  $\lambda$  to pass through
- Frequency  $\nu$  – number of wavelengths passing through a fixed point in one second
- $\nu = \frac{1}{\tau}$      $\nu = \frac{c'}{\lambda}$      $c' = \frac{c}{n}$      $n =$  refractive index
- Refractive index can usually be assumed to be constant, but varies across wavelengths.

### Converting frequency and wavelength differences

- Difficult to convert a small difference in frequency into a difference in wavelength
- Linear approximation for small differences:  $\Delta\nu = -\frac{c}{\lambda^2} \Delta\lambda$

### Homework problems

2. A laser light field is modelled by the following expression,

$$E(t, x) = A \sin\left(2\pi\nu t - \frac{2\pi\nu x}{\lambda} + \phi\right) .$$

a) Plot the expression as a function of  $x$  and for values of  $t = (0, 0.1/\nu, 0.2/\nu$  and  $0.3/\nu)$ . Let the amplitude  $A = 1$  and the phase offset  $\phi = 0$ . Which way is the wave moving?

b) Now shift the phase of the wave such that  $\phi = \pi/2$ . How do the zero crossings line up with the original position of the function i.e. When  $\phi = 0$ ?

c) The light wave defined by the function enters a material with refractive index of  $n = 1.5$ . Plot the expression as a function of  $x$  outside the material, just inside the material and as it exits. It is easier if you place the interfaces at the zeroes of the wave.

3. The emission of one laser is at  $\lambda = 632\text{nm}$  and has a linewidth of  $\Delta\nu = 1$  MHz. A second laser lases at  $\lambda = 640\text{nm}$  and has a linewidth of  $\Delta\nu = 30\text{GHz}$ . Yet another source lases at  $\lambda = 1.5\mu\text{m}$  also has a linewidth of  $\Delta\nu = 30\text{GHz}$ . Convert each of these linewidths into units of nanometres.

**Optical path length**

- Light travels through a medium, speed is retarded and wavelength reduced by  $\frac{1}{n}$
- Optical path length  $OPL = nL$
- Phase is given by  $\phi = \frac{2\pi L}{\lambda/n}$

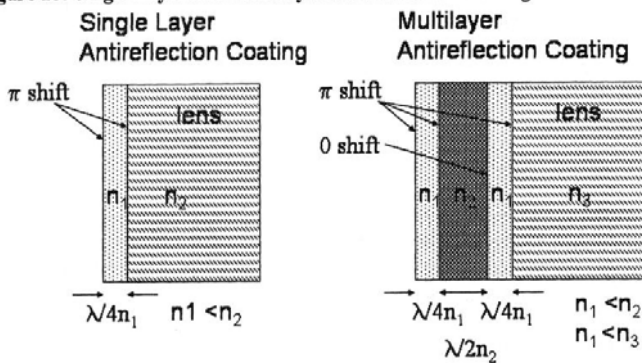
**Interference of light waves – principle of superposition**

- Interference – two light waves adds up
  - Destructive interference – two light waves add to zero
  - Constructive interference – waves add up to twice the size

**Destructive interference of light at the CD surface**

- Optical Path Length travelled by the reflected wave has destructive interference when  $\frac{\lambda}{2} = 2H_B n$  or  $H_B = \frac{\lambda}{4n}$

Figure 18: Single Layer and Multilayer Antireflection Coatings



**Homework Problems**

4. A ray of light, with wavelength  $\lambda = 500\text{nm}$  in air, is travelling in a material of  $n_2 = 1.5$ . It encounters a thin layer of material with refractive index  $n_1 = 2.2$  followed by air as shown in the figure below.

- a)
- b)
- 5.
- a)
- b)

## Lens systems and non-ideal behaviour

### Snell's law and refraction

- Describes refraction of light at an interface between different materials
- $n_1 \sin \theta_1 = n_2 \sin \theta_2$

### Lens equation and simple results

- Simple lens equation
 
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$
  - $f$  – focal length of the lens
  - $s$  – distance to the object
  - $s'$  – distance to the image
- If the object is at an infinite distance away ( $s = \infty$ ), then the image is at a distance equal to the focal point.

### Homework Problem

6. A lens with focal length +100 mm is used to image a ten cent coin. How far should the coin be placed from the lens to cause an apparent inflation by a factor of two?

### Aberrations – non ideal lens behaviour

- Chromatic aberration – caused by different refractive indices for different wavelengths
  - But laser light is monochromatic
- Spherical aberration
  - Lenses have spherical surfaces as a result of manufacturing process
  - There is not one focal point
  - Rays form a minimum diameter called the circle of least confusion
  - $Error \propto x^3$ ,  $x$  is distance from the axis
  - Corrected using aspherical lens, which has a complex shape

### Homework Problem

7. A person who is slightly near sighted (can see well at close distances) looks into the distance without glasses and says that they can see a neon light with red lettering perfectly but sees another sign with blue lettering as fuzzy.

- Draw a diagram to explain how this can be.
- Is wearing red coloured glasses a cure for nearsightedness?

### Astigmatism

- Astigmatic lens – shaped in such a way that it has different focal lengths for each of the two orthogonal planes of a light beam.
- e.g. Cylindrical lens – focuses onto a line rather than a spot

- In CD player, the distance between reading head and CD  $\approx 1\mu\text{m}$ .
- Uses astigmatism to identify whether it is too far or too close.

## Diffraction – propagation of light

### The wavelike nature sets some limits on imaging with light

- Light is a wave, so it does not form sharp shadows
- Light shone through two closely spaced holes form spacial oscillations rather than two points.

### Huygen's principle – superposition of individual wavelets

- Huygen's principle – every point on the primary wavefront of a light wave is a source of spherical secondary wavelets.
  - Wavelets add to form the primary wavefront
  - Wavelets have same speed and frequency

### Huygen's principle and a plane wave truncated by a single slit

- The emitted light may be represented as a single wavelet, and does not form a well defined spot with same shape as the slit
- As the slit is widened, multiple wavelets interfere to create light and dark patches that are closer together, until a point when the resultant pattern looks similar to the slit size.
- Single slit diffraction pattern geometry
  - (Figure 35)
  - Path difference  $\Delta L = a \sin \theta$
  - Constructive interference when path difference are multiples of the wavelength  
 $a \sin \theta = m \lambda$

### Huygens and two wavelets – the double slit experiment

- Path length difference for a bright patch:  $D_1 - D_2 = \lambda m$

### Homework Problem

8. For a wavelength of 780nm and a hole spacing of  $6\mu\text{m}$ , determine the distance from the axis to the first bright patch if the screen is 100nm away

9. A thin piece of glass is placed in one of the slits such that one of the slits is radiating half a wavelength behind the other (a  $\pi$  phase shift).

a) Redraw the two slit experiment described previously but what happens to the oscillatory pattern of light on the screen? The thin piece of glass is replaced by a wedge of glass so that the thickness of the glass covering the hole may be varied to change the phase in a continuous manner.

b) Consider what happens when the phase of one of the wave emitted waves is varied in a continuous and linear fashion with time from 0 to many multiples of  $\pi$ . What happens to the pattern on screen?

### The airy pattern and airy disk

- When parallel light is focused onto a spot, even by a perfect lens, the focused spot is a pattern of concentric rings called “Airy pattern”
- The diameter of the central intensity spot (The Airy Ring), is given by  $d = 1.22 \frac{\lambda}{\sin \theta}$
- This approximates to  $d = 1.22 \frac{\lambda f}{D}$

### Homework Problem

10. A spy satellite is orbiting the earth in a LEO (low earth orbit), 100km above sea level. A camera with a telescopic lens is pointing down at Sydney University, calculate the diameter of the telescope's aperture to enable the telescope to see two separate people standing 50 cm apart. Assume they are separated by the Airy disk and the central wavelength is in the green  $\lambda = 500\text{nm}$ .

### The diffraction grating and the tracking mechanism

- Diffraction grating splits the main laser beam into two additional beams ( $\pm 1^{\text{st}}$  order diffracted beam) to move along the land, and maintain the difference between the intensities of those beams.
- Uses the same formula as double slits, but 'a' refers to the spacing between the diffraction grating rules.

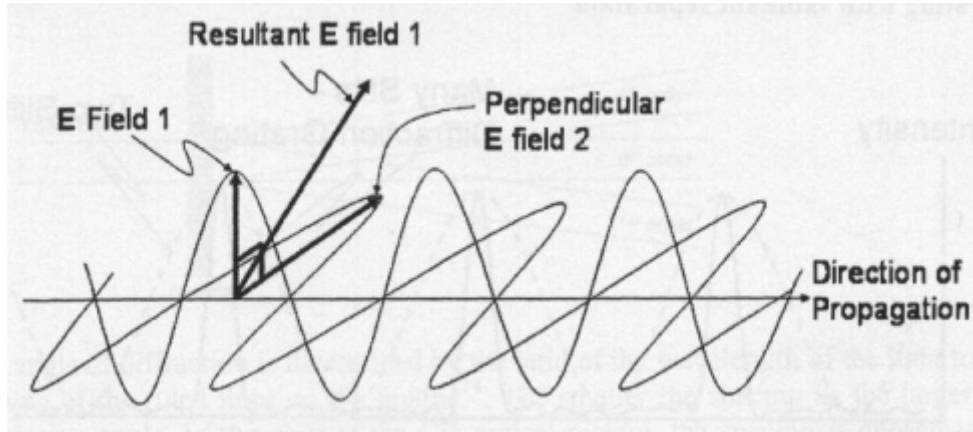
### Homework Problem

11.

- a) Design a diffraction grating such that the first diffraction orders are separated from the  $0^{\text{th}}$  order by 10cm on a screen 10cm away from the grating. The wavelength of the illuminating parallel laser light is  $\lambda = 632\text{nm}$ .
- b) If a second laser at  $\lambda = 316\text{nm}$  is shone through this grating where do the diffraction orders fall on the screen?

## Polarisation of light

### The two orthogonal polarisation states of light



- Decompose light into two orthogonal polarisation directions perp'r to direction of propagation.
- In Birefringent materials, light travelling in one polarisation travels slower than in the other polarisation; that is, it has a different refractive index for each polarisation direction.
  - Birefringence can occur naturally due to crystal structure or caused by stress or applied fields.
- Align the crystal to create a spatial separation of the orthogonally polarised beams.
  - E.g. obliquely aligned glass reflects 's' (perpendicular) polarisation, but transmits 'p' polarisation

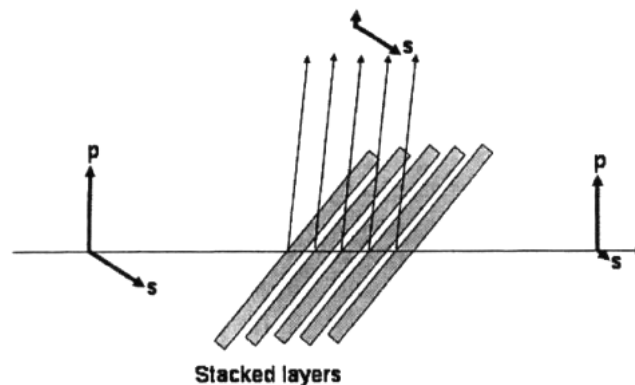
**Brewster's angle**

- An angle of incidence where p-polarised light will be completely transmitted without loss, while s-polarised component is strongly reflected.
- $\tan(\theta_B) = \frac{n_t}{n_i}$   $n_i$  = incident medium refractive index;  $n_t$  = transmitted medium refracted index

**Homework Problem**

12. Show that, at Brewster's angle, the reflected beam and the transmitted beam are at right angles to each other. Hint: Use Brewster's angle equation and apply Snell's law.

**The polarisation beam splitter**



- Reflects almost all of one polarisation, which the other is nearly all transmitted.

**Changing the polarization of the light**

- Phase delay of a polarisation in birefringent material causes resultant polarisation vector to change.

- $\frac{1}{2}$  wave plate – lags one polarisation by  $\frac{1}{2} \lambda$ .
- This leads to the polarisation flipping about the fast axis of the crystal.
- Lagging by  $\frac{1}{4}$  wave is called circularly polarised.

### The $\frac{1}{4}$ wave plate in the CD pick-up

- Diode laser – linearly polarised light
- Before hitting surface of disk -  $\frac{1}{4}$  wave plate
- Reflected on surface of disk – circularly polarised
- After hitting surface of disk -  $\frac{1}{4}$  wave plate; creates linearly polarised light from circular polarise light
- Light at the end is linearly polarised but rotated  $90^\circ$ .

### Homework Problem

13. Light at 400nm which is linearly polarised in the vertical direction passes through a birefringent crystal with the fast axis oriented at  $45^\circ$  to the direction of the polarisation of the light. If the birefringence, the difference in the refractive index for each axis, is  $(n_f - n_s) = 10^{-4}$  calculate the length of the crystal required to create circularly polarised light.

14. Austin Powers wishes to send a secret communication to a submarine lurking below the surface of very still water fjord. It is best if no light is reflected since this can be detected by the forces of evil. Draw a diagram showing the polarisation and angle of incidence he should use. Remember the index of refraction for water is  $n = 1.33$ .

### Putting it all together

- CD system: 780nm 1mW laser
- Passes through diffraction grating to create two additional beams
- Beams made parallel by collimating lens
- Light focused onto small spot on CD using aspherical lens, size limited by lens aperture size
- Height of bumps on surface is such that when a portion of beam is on bump and a portion on land, then there would be destructive interference between the two reflected component beams.
- Reflected light passes through  $\frac{1}{4}$  wave plate, turning it into linear polarised light

## Part 2 – Communication System Components

### Overview of an optical communications network

- Modulation and detection of optical signals
- Light source (laser)
- Medium – optical fibres and waveguides
- Multiplexing and demultiplexing (TDM, WDM)
- Signal amplifier
- Compensator for non-ideal behaviour

## Optical fibres

### Why optical fibre?

- High bandwidth
- Lower attenuation

### Bandwidth

- Carrying information by modulating a wave
  - Modulating – encode an existing detectable carrier signal
- Creating periodic waveforms from sine waves
  - Pure note (single frequency waveform) has a single spike in spectral domain
  - Superpose harmonics over the fundamental
  - Any periodic waveform may be decomposed into a specific summation series of sinusoidal waveforms, each with a particular frequency and amplitude. (Fourier series)
- Creating non-periodic waveforms from sine waves
  - If the signal is aperiodic, then the spectrum would be a continuum of frequency components rather than a series of discrete components
  - Gaussian pulse generates a Gaussian spectrum
  - Every temporal waveform has an associated spectrum
  - The shorter the pulse the wider the spectral bandwidth
- Relationship between bandwidth and speed of information flow
  - Shorten pulses to increase amount of information per second
  - But spectrum becomes wider (covers more bandwidth)  $\Delta \nu_{BW} \approx \frac{1}{\Delta \tau}$
- Spectrum of a modulated carrier wave
  - Carrier signal – already at high frequency
  - Modulated within a small fraction of carrier frequency
  - Modulating a carrier signal causes a spread in the spectrum

### Homework Problem

15. Consider the Fourier series 
$$f(x) = \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^2} \sin\left(\frac{n\pi x}{L}\right)$$

a. Write down and plot the first three terms in this series as function of  $x$  with  $L = 1$  on the same graph for two periods of the fundamental, noting that  $n$  is odd i.e.  $n = 1, 3, 5, \dots$

b. Plot the sum of these components i.e. superpose them. What waveform does this series represent?

c. Plot the spectrum of this waveform in the spectral domain.

d. Consider the wave represented by the series above. What happens to the form of the wave as the bandwidth is reduced?

16. A pulse is  $\Delta \tau = 10\text{ps}$  long. Give an estimate of the bandwidth.

17. A company wishes to an amplifier for optical pulses at  $\lambda = 590\text{nm}$  and duration  $\Delta \tau = 150\text{fs}$  long (1

femtosecond =  $10^{-15}$  s). How wide must the spectrum of the gain medium be to amplify these pulses without lengthening them? Express your answer in units of wavelength [nm].

### Loss or Attenuation

- Attenuation coefficient  $\alpha$  measured in decibels per unit length (dB/km)
- $\alpha = \frac{1}{L} 10 \log \left( \frac{P(0)}{P(L)} \right)$
- $P(0)$  is initial power,  $P(L)$  is power after the signal has been transmitted over L km.
- Then, the power after a distance z is  $\frac{P(z)}{P(0)} = 10^{-0.1\alpha z}$

### Homework Problem

18. A signal carried by a medium is 10% of its original strength after 2km.

- What is the attenuation in dB/100km of the carrier medium?
- What is the signal strength after 1km?
- What is the signal strength after 100km?

### Electromagnetic radiation as a carrier – a short history

- Electromagnetic radiation is much better to transmit signals than sound
- Light was known to be a great carrier of information at 100THz
- But there was no cheap bright, coherent and monochromatic source (diode laser), and light guides
- Cheap glass fibres changed it all.

### Total internal reflection

- Light angle exceeds a critical angle derived from Snell's law, given by  $\sin(\theta_c) = \frac{n_2}{n_1}$
- Works when light is going from medium with high to medium with lower refractive index.
- Total internal reflection is entirely lossless

### Homework Problem

19. Calculate the critical angle for light leaving a medium with refractive index  $n = 1.33$  into air.

### Evanescient fields

- Light is a wave, and not a well defined ray.
- Instead of abruptly turning at the interface, light field exists for a short distance on the other side
- Distance is on the order of wavelength of light
- Coupling – substances interact with the evanescent field, causing scattering.

### Guiding in an optical fibre

- TIR allows light above critical angle of incidence to be guided losslessly.

- Single mode fibre – fibre has a core sized near wavelength of light
- Polymer jacket protects silica core and couples out any light propagating in the cladding

### Homework Problem

20. Consider two identical simple optical fibres which guide light by TIR at their outside interfaces with air. Let one fibre be guiding light and the other empty or dark. Describe what you think might happen if the two optical fibres, which are parallel to each other, are brought into contact with each other.

21. In the modern optical fibre, most of the light is in the core. Can you think of ways in which we might modify sections of the fibre in order to couple light out of the core?

### Numerical aperture

- Numerical aperture – the acceptance angle for rays going into or out of the fibre.
- Given by sine of acceptance angle  $NA = \sin(\theta)$
- $\theta$  is the half angle of cone of rays which can be totally internally reflected
- High NA = large acceptance angle
  - Core dimensions of the fibre must be reduced for a single mode to be maintained

### Homework Problem

22. Consider a fibre with cladding index of  $n = 1.5$  and an index contrast of 3%.

- Calculate the numerical aperture of the fibre
- Which part of the fibre has the higher index; the core or the cladding? Why?

23. Show that  $NA = (n_2^2 - n_1^2)^{1/2}$  can be simplified to  $NA = (2n_1\Delta)^{1/2}$ , where  $\Delta = n_2 - n_1$  and is small compared to the index  $\Delta \leq n_1$ .

### Making an optical fibre

- Hollow silica preform is heated then drawn

### Losses in optical fibres

- Material loss
  - Rayleigh scattering – from imperfections  $attenuation \propto \frac{1}{\lambda^4}$
  - UV absorption
  - Infrared absorption
- Form factors and bend loss
  - Bend loss – bending allows light to leak out of the core; function of radius and wavelength
  - Longer wavelengths are more sensitive to bends
- Coupling loss
  - Fresnel reflection – reflection at the end of a fibre; reduced by fusing ends together
  - Spatial overlap – fibre ends cores must overlap spatially

- Angular overlap – guiding cores in each fibre section must be parallel

### Temporary connectors

- Photonic connectors brings two fibre sections close together
- Evanescent wave extends across interface
- Eliminate Fresnel reflections

### Dispersion

- What is dispersion?
  - Original signal pulse stretches out in time
- Modal dispersion
  - Some trajectories take longer to propagate through the fibre
  - Longer path = slower pulse
- Chromatic dispersion
  - Refractive index depends on wavelength
  - Material dispersion – material's electron resonance causes phase lag, and is a dispersive effect
  - Waveguide dispersion – SMF fibres only, associated with geometry of core, relevant to wavelength of light.

### Homework Problem

24. Two rays are launched into an optical fibre. One ray is launched at  $10^\circ$  to the axis of the fibre and continues to bounce down the fibre at this angle in one plane. The other ray is along the axis. A pulse of  $\Delta\tau = 10\text{ps}$  in length is launched into the fibre such that half of the energy is in each ray. The index of refraction is  $n = 1.5$ .

- What happens to the pulse as it progresses along the fibre?
- If we wish to build a communications system using this fibre, is there a maximum distance for transmission (assuming no losses). If so, how long is it?

### Chromatic dispersion and pulse propagation

- One end of pulse's spectrum travels at a different speed to the other
- Spreading of initial pulse as a result of dispersion:  $\Delta T = D_T \Delta\lambda L$
- $D_T$  is dispersion in ps/nm/km,  $\Delta\lambda$  is spectral spread,  $L$  is length of fibre
- $$\Delta\lambda = \frac{\lambda^2}{c} \frac{1}{\Delta\tau}$$

### Homework Problem

25. Calculate the pulse width of a  $\Delta\tau = 10\text{ps}$  pulse after it has travelled 100km in an optical fibre with dispersion  $D_T = 20\text{ps/nm/km}$ . The wavelength is  $1.5\mu\text{m}$ .

## Modes

- What is a mode?
  - A natural vibration mode
- Modes of a fibre
  - Ways in which light may propagate in the fibre
  - Multi Mode Fibre – number of transverse modes of propagation is a function of wavelength
- Normalised frequency – supported modes
  - Normalised frequency  $V = 2\pi \frac{a}{\lambda} NA$ ,  $a$  is core radius
  - Fibre is single mode when  $V < 2.405$
  - If  $a \gg \lambda$  or NA increased, the fibre may support more modes.
  - Number of modes  $M = \frac{4}{\pi^2} V^2$

## Homework Problem

26. A fibre with cladding index  $n = 1.5$  and index contrast of 0.5% has a core diameter of  $d = 4 \mu\text{m}$ . Calculate the wavelength at which a second mode appears.

## Gratings

### History

- Discovery: intense light forming standing wave pattern in fibre, changing the refractive index
- Creates a periodic structure over a long length

### Applications

- Gratings can be used as filters
- Gratings are sensitive to strain and temperature, so may be used as sensors

### The optical fibre Bragg grating

- Has a periodic refractive index modulation
- Gives constructive interference when the light reflected from one bump is in phase with the light reflected from another. Bragg condition:  $2n\Lambda = \lambda_B$ ,  $\lambda_B$  is Bragg wavelength, Grating spacing  $\Lambda$
- Ratio of width of the central peak (in reflection vs wavelength) to reflected wavelength is the same as the index modulation depth to the total index:  $\Delta\lambda_{BW} = \lambda_B \frac{\Delta n}{n}$

### Sidelobes and apodisation

- Sidelobes – on either side of the central peak
- Due to the fact that the grating starts and ends abruptly.
- The spacing between minima is linearly dependent on length of grating. Longer = closer sidelobes.
- Apodised grating – smoothing the edges of the grating
  - Refractive index modulation is ramped up and down slowly

### The coupling coefficient and grating strength

- Strength of a uniform grating
  - Coupling coefficient  $\kappa_{ac} = \pi \frac{\Delta n}{\lambda}$  ( $\text{cm}^{-1}$ ) relating to the grating strength
  - Reflectivity of grating  $R = \tanh^2(\kappa_{ac} L)$ , L is length of grating
- Strength on a dB scale
  - $R = 10 \log(R)$  dB,  $T = 10 \log(1 - R)$  dB
- Bandwidth of weak gratings ( $\kappa_{ac} L < 3$ )
  - $\Delta \lambda_{BW} \approx \frac{\lambda^2}{nL}$
  - Short gratings have wide bandwidths and long gratings have narrow bandwidths, so long as they are weak.
- Bandwidth of strong gratings ( $\kappa_{ac} L > 3$ )
  - $\Delta \lambda_{BW} \approx \frac{\lambda^2}{\pi} \kappa_{ac}$
  - Bandwidth does not vary according to physical length of the grating because the light is completely reflected early in the grating and does not see the additional grating length.

### Homework Problem

27. A uniform grating at  $\lambda_B = 1550\text{nm}$  has a refractive index modulation of  $\Delta n = 3 \times 10^{-5}$  on  $n = 1.5$  and is 50mm long.

- a) What is the reflectivity of this grating? Express this as transmission in dB.
- b) What is the underlying actual physical period length of the grating?

28. A grating is required at  $\lambda = 1550\text{nm}$  with a reflectivity of 80% and  $\Delta \lambda_{BW} = 20\text{pm}$  wide in a material with  $n = 1.5$ . Calculate the length of the grating.

29. A grating is written  $\lambda_B = 1552\text{nm}$  for which the reflectivity is 99.5%. On measuring the bandwidth of the grating, it is found to be 10pm wide.  $n = 1.5$ . What is Kappa for this grating? How long is the grating?

### The grating as a sensor

- Grating period (and resonant wavelength) is sensitive to strain and temperature which changes the refractive index.

### Homework Problem

30. The resonance of a grating located in a support beam of a building has shifted by 4.5nm in the last three weeks. Assuming that the temperature has remained constant and that the change in wavelength around  $\lambda_B = 1550\text{nm}$  is entirely due to stress, calculate the increase in stress in the beam.

31. Specifications for gratings used as filters in telecommunications require that they are measured with a

precision and accuracy of 1pm. How precisely must the fibre stress and temperature be controlled?

### The grating as a dispersion compensator

- Uses chirped grating. The period of the grating changes along the length.
- Reflected light experiences a different delay depending on its wavelength.
- The delay between two points on the grating is given by  $\Delta\tau = \frac{2nL}{c}$
- May be used as a dispersion compensator, to undo the wavelength dependent delay caused by dispersion in a long length of optical fibre.

### Homework Problem

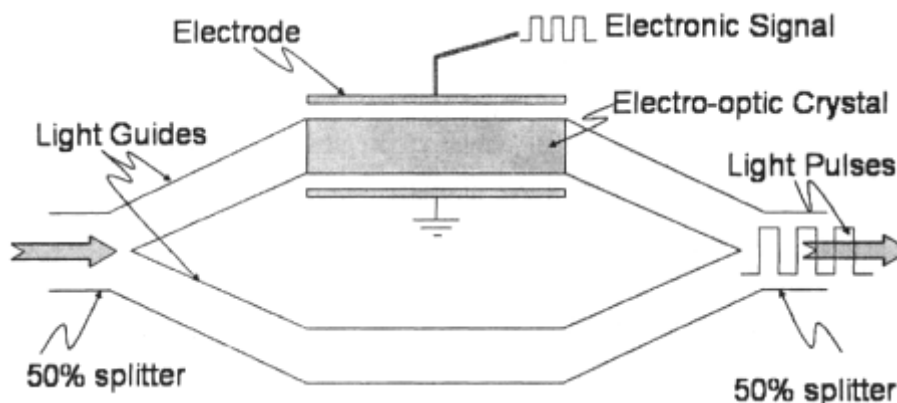
32. A train of pulses at  $\lambda=1550\text{nm}$ , each  $\Delta\tau = 10\text{ps}$  long, propagates down a fibre link 100km long with  $n=1.5$

- Calculate the bandwidth of the communications channel both as a frequency and as a wavelength.
- A chirped grating is used to compensate for the dispersion created by the link. Assume the dispersion of the fibre is  $D_T = 17\text{ps/nm/km}$ . What is the dispersion of the grating required to compensate this link? How long will this grating be?
- If the blue wavelengths are delayed mostly by the fibre link, which end of the grating should be the blue end? The one at the end the light enters or the one furthest away?

## Modulation: Interferometers & electro-optics

### The Mach Zehnder modulator

- Converts electrical signals to optical signals
  - Uses electro-optic effect to modify refractive index of a material to cause interference



### The interferometer

- Uses interference to measure quantities which affect the phase of a light beam
- Light is split, then recombined.

- Electric field is given by  $E = \sin\left(2\pi\nu t - \frac{2\pi}{\lambda}x + \phi\right)$ ,  $\phi = 2\pi\frac{D}{\lambda}$ , D is optical path length
- If one arm's OPL is made to change, this can be detected by interference.
- Resultant electric field is given by  $E_R = \sin\left(2\pi\nu t - \frac{2\pi}{\lambda}x + \Delta\phi\right) + \sin\left(2\pi\nu t - \frac{2\pi}{\lambda}x\right)$ 
  - $E_R = 2\cos\left(\frac{\Delta\phi}{2}\right)\sin\left(2\pi\nu t - \frac{2\pi}{\lambda}x + \frac{\Delta\phi}{2}\right)$
  - The cosine term modulates the amplitude of the light field as a function of phase difference
- Light intensity of the resultant field – take time averaged square  $\frac{I_R}{I_{bright}} = \cos^2\left(\frac{\Delta\phi}{2}\right)$

### Homework Problem

33. A laser at wavelength  $\lambda = 632\text{nm}$  is used as a light source for an interferometer. A piezoelectric material (expands when a potential is applied) with an expansion coefficient of  $1\mu\text{m}$  per  $10\text{V}$  is used to drive a phase shifter for the laser beam. For every  $1\mu\text{m}$  of piezo expansion, the optical path length changes by  $333\text{nm}$ .

a) Assuming the interferometer is initially on a bright fringe, calculate the output intensity as a ratio to the bright fringe intensity when a potential of  $14\text{V}$  is applied to the piezo.

b) Assume the piezo has an expansion coefficient of  $\frac{\Delta L}{\Delta T} \frac{1}{L} = 1 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$  and the length of piezo is  $10\text{mm}$ .

i) what is the expansion of the material in  $\text{nm}$  for a temperature change of  $5^\circ\text{C}$ ?

ii) Calculate the change in the output intensity for the interferometer as a ratio of a bright fringe if the interferometer path difference was initially set such that the output was  $0.5$  of a bright fringe. Don't forget the path conversion factor of  $3:1$  as before.

### Common mode rejection

- Construct system such that disturbance or perturbation affects both path lengths in the same way
  - e.g. Fluctuations in air pressure, temperature, thermal expansion
- Ideal to have both paths trace out the same route
- In MZ modulator, noise is rejected by keeping the device small
  - Less distance for any perturbation to act over
  - Both paths likely to experience the same disturbance

### Temporal coherence

- Coherence – the ability of waves to interfere with one another
- Temporal coherence – a section of wave at one time can interfere with with wave at another time
  - Sine wave is perfectly temporally coherent
- As time period separating the two sections of wave is shortened, it becomes more coherent
- Coherence length – the distance the light travels in the coherence time

- Coherence length is related to bandwidth of source.  $\Delta L_{coh} \approx \frac{c}{\Delta \nu_{BW}}$

### Homework Problem

34. A laser source has bandwidth of 30GHz.

- Calculate the coherence length of the source.
- What is the time separation of sections of the light wave that can still interfere?
- What is the maximum path length difference that an interferometer, built with this source, can have?

35. An interferometer path length difference is varied from zero. The fringes are no longer visible after 100 $\mu$ m. What is the spectral width of the light source (yellow or around 490nm) in frequency units and in nanometers?

### The electro-optic effect

- In general, since the electric field acts only along one axis, then only one polarisation of light sees the changed refractive index, depending on the crystal structure and orientation.
- Change in refractive index can be expressed as  $\Delta n(t) = n_0^3 K_p E(t)$   $K_p$  is the Pockels constant

### Other ways of changing light

- Magnetic field can influence electrons similar to electric field, but generally not used since a large magnetic field is required and is hard to switch rapidly
- High frequency acoustic wave can create a diffraction grating
- Nonlinear optics

### Phase matching

- Phase matching – making sure that two waves travel at the same speed so that they are in phase
- E.g. If crystal is made too long for one driving frequency, then the the modulating effects may cancel and achieve no modulation.

### Homework Problem

36. A modulator is required to modulate light at 40GHz.

- What is the maximum length that the modulator can have to do this task if the refractive index is  $n = 2.2$
- Assuming that this length of crystal is not sufficient to create a  $\pi$  phase shift at the maximum achievable electric field strength, can you think of a more sophisticated electrode arrangement which allows the driving field and the propagating light to stay in phase over a longer length?